THE TEMPERATURE CALCULATION FOR A PLATE AND CYLINDER HEATED SIMULTANEOUSLY BY RADIATION AND CONVECTION

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A solution is derived for the Fourier equation by the method of averaging the functional correction factors in the case of convection–radiation heating. Tables are presented for the practical calculation of plate and cylinder temperatures.

The problem of convection–radiation heating reduces to the solution of the heat-conduction equation

$$\frac{\partial^2\theta}{\partial\xi^2}+\frac{m}{\xi}\frac{\partial\theta}{\partial\xi}=\frac{\partial\theta}{\partial\tau} \tag{1}$$

with the boundary condition

$$\left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=\rho}=[1-\theta_p^4(\tau)]+\eta[1-\theta_p(\tau)], \tag{2}$$

to which is added, in the case of inertial heating ($0 \le \tau \le \tau_0$), the conditions

$$\theta_1(\xi,\tau)|_{\xi=\beta(\tau)}=\theta_0=\text{const}, \tag{3}$$

$$\left.\frac{\partial\theta_1}{\partial\xi}\right|_{\xi=\beta(\tau)}=0, \tag{4}$$

$$\theta_1(\xi,0)=\theta_0, \tag{5}$$

while in the case of regular heating ($\tau_0 \le \tau < \infty$):

$$\left.\frac{\partial\theta_2}{\partial\xi}\right|_{\xi=0}=0, \qquad \theta_{2p}(\tau_0)=\theta_p(\tau_0). \tag{6}$$

Research such as [1-3] is based on the use of a computer, which is not always a practical approach.

The method of averaging the functional correction factors [4, 5] yielded the following approximate analytical solution for the stated problem (1)-(6) in the assumption that

$$T_m=\text{const},\quad T_0=0,\quad \lambda=\text{const},\quad \sigma_a=\text{const},\quad \alpha_{con}=\text{const}. \tag{7}$$

The inertial heating stage

$$\theta_1(\xi,\tau)=\frac{\theta_{1p}(\tau)}{[\rho-\beta(\tau)]^2}[\xi-\beta(\tau)]^2\quad(\beta(\tau)\leqslant\xi\leqslant\rho), \tag{8}$$

$$\theta_{1p}(\tau)=\frac{\eta\varkappa_1[\rho-\beta(\tau)]}{\varkappa_1-1+\eta\varkappa_1[\rho-\beta(\tau)]}, \tag{9}$$

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TABLE 1. Values of the Relative Surface Temperature $\theta_{1p}(\tau_0)$ at the Conclusion of the Inertial Heating Stage

η	Sk											
	0,25		0,50		0,75		1,00		1,50		2,00	
	m=0	m=1	m=0	m=1	m=0	m=1	m=0	m=1	m=0	m=1	m=0	m=1
0,0	0,102	0,102	0,314	0,314	0,506	0,506	0,645	0,645	0,805	0,805	0,880	0,880
0,2	0,121	0,107	0,335	0,306	0,523	0,500	0,659	0,612	0,810	0,780	0,881	0,860
0,4	0,141	0,126	0,356	0,323	0,540	0,500	0,668	0,631	0,812	0,790	0,884	0,861
0,6	0,154	0,144	0,360	0,344	0,556	0,519	0,679	0,643	0,816	0,798	0,886	0,870
0,8	0,177	0,170	0,396	0,366	0,569	0,536	0,689	0,655	0,823	0,800	0,890	0,880
1,0	0,193	0,180	0,414	0,385	0,587	0,552	0,699	0,666	0,830	0,803	0,893	0,875
1,5	0,232	0,220	0,457	0,428	0,617	0,586	0,720	0,694	0,840	0,819	0,898	0,884
2,0	0,267	0,256	0,489	0,468	0,638	0,614	0,739	0,712	0,846	0,830	0,901	0,891

$$\theta_{1p}(\tau_0) = \frac{\eta \varkappa_1 \mathrm{Sk}}{\eta \varkappa_1 \mathrm{Sk} + 2(\varkappa_1 - 1)}, \tag{10}$$

$$\rho - \beta(\tau) = 2\sqrt{3(1+m)\tau}, \tag{11}$$

$$\tau_0 = \frac{\rho^2}{12(1+m)}, \tag{12}$$

$$\varkappa_1 = 1 + \frac{0.275 + 0.058m}{\mathrm{Sk}}\,\eta. \tag{13}$$

The regular heating stage

$$\theta_2(\xi, \tau) = \theta_{2p}(\tau) - \frac{\mathrm{Sk}}{2}\varkappa(\theta_{2p}\ \eta)\left[1 - \theta_{2p}^4(\tau)\right]\left(1 - \frac{\xi^2}{\rho^2}\right). \tag{14}$$

$$\theta_2(0, \tau) = \theta_{2c}(\tau) = \theta_{2p}(\tau) - \frac{\mathrm{Sk}}{2}\varkappa(\theta_{2p}\ \eta)\left[1 - \theta_{2p}^4(\tau)\right]. \tag{15}$$

The relative surface temperature is determined from the transcendental equation

$$(1+m)\,\mathrm{Sk}\,(\mathrm{Fo} - \mathrm{Fo}^0) = (1+\eta)^{-1}\left[\varphi(\tau) - \varphi(\tau_0)\right] + \frac{\mathrm{Sk}}{3}\left[\psi(\tau) - \psi(\tau_0)\right] + \frac{\mathrm{Sk}}{3}\left[\omega(\tau) - \omega(\tau_0)\right] + (1+\eta)^{-1}\left[\Psi(\tau) - \Psi(\tau_0)\right] = \Omega(\tau) - \Omega(\tau_0), \tag{16}$$

where

$$\Omega(\tau) = (1+\eta)^{-1}\varphi(\tau) + \frac{\mathrm{Sk}}{3}\left[\psi(\tau) + \omega(\tau)\right] + (1+\eta)^{-1}\Psi(\tau); \tag{17}$$

$$\varphi(\tau) = \frac{1}{2}\left[\operatorname{arctg}\theta_{2p}(\tau) + \operatorname{arth}\theta_{2p}(\tau)\right]; \tag{18}$$

$$\psi(\tau) = -\ln\left[1 - \theta_{2p}^4(\tau)\right]; \tag{19}$$

$$\omega(\tau) = -\ln\left\{1 + \eta\left[1 - 0{,}75\theta_{2p}(\tau)\right]\right\}; \tag{20}$$

$$\Psi(\tau) = \varepsilon\nu_1(\tau) + \varepsilon^2\nu_2(\tau) + \varepsilon^3\left[\nu_3(\tau) - \omega(\tau)\right]; \tag{21}$$

$$\nu_1(\tau) = -\frac{1}{4}\ln\frac{1 - \theta_{2p}^2(\tau)}{1 + \theta_{2p}^2(\tau)}; \tag{22}$$

$$\nu_2(\tau) = -\frac{1}{2}\left[\operatorname{arctg}\theta_{2p}(\tau) - \operatorname{arth}\theta_{2p}(\tau)\right]; \tag{23}$$

$$\nu_3(\tau) = -\frac{1}{4}\ln\left[1 - \theta_{2p}^4(\tau)\right]; \tag{24}$$

$$\varepsilon = \frac{0.75\eta}{1+\eta}; \tag{25}$$

TABLE 2. Values of the Functions $\Omega(\theta_p, \text{Sk}, \eta)$

Sk	θ_p	η 0	0,2	0,4	0,6	0,8	1,0	1,5	2,0
0,25	0,100	0,100	0,071	0,051	0,035	0,025	0,017	—	—
	0,200	0,200	0,162	0,132	0,111	0,093	0,080	0,055	0,038
	0,300	0,299	0,256	0,221	0,192	0,170	0,151	0,116	0,089
	0,400	0,398	0,355	0,315	0,282	0,254	0,230	0,180	0,150
	0,500	0,501	0,461	0,418	0,381	0,347	0,318	0,261	0,218
	0,600	0,604	0,573	0,530	0,490	0,452	0,417	0,347	0,293
	0,700	0,716	0,698	0,658	0,617	0,572	0,533	0,448	0,382
	0,800	0,842	0,849	0,816	0,771	0,721	0,676	0,574	0,492
	0,900	1,014	1,104	1,043	1,001	0,944	0,898	0,760	0,653
	0,999	1,776	2,121	2,221	2,178	2,142	1,977	1,775	1,443
0,50	0,300	0,299	0,244	0,198	0,160	0,129	0,103	—	—
	0,400	0,397	0,342	0,292	0,251	0,214	0,187	0,118	0,075
	0,500	0,495	0,446	0,394	0,349	0,308	0,273	0,201	0,145
	0,600	0,593	0,553	0,502	0,455	0,410	0,369	0,285	0,220
	0,700	0,693	0,677	0,621	0,574	0,522	0,478	0,381	0,303
	0,800	0,798	0,778	0,760	0,709	0,654	0,604	0,491	0,399
	0,900	0,925	0,969	0,945	0,897	0,836	0,786	0,638	0,569
	0,999	1,260	1,601	1,697	1,651	1,611	1,443	1,233	0,894
0,75	0,500	0,490	0,431	0,371	0,317	0,269	0,227	0,140	—
	0,600	0,581	0,533	0,474	0,419	0,368	0,323	0,224	0,146
	0,700	0,671	0,640	0,584	0,530	0,472	0,422	0,313	0,226
	0,800	0,755	0,748	0,704	0,647	0,587	0,522	0,408	0,306
	0,900	0,837	0,875	0,826	0,794	0,728	0,674	0,517	0,438
	0,999	0,944	1,081	1,173	1,072	1,079	0,908	0,690	0,544
1,00	0,600	0,570	0,511	0,446	0,384	0,326	0,273	0,162	—
	0,700	0,648	0,607	0,546	0,486	0,423	0,366	0,246	0,146
	0,800	0,711	0,698	0,648	0,586	0,520	0,460	0,325	0,223
	0,900	0,748	0,782	0,748	0,691	0,620	0,602	0,395	0,308
1,50	0,700	0,602	0,546	0,472	0,399	—	—	—	—
	0,800	0,626	0,598	0,536	0,473	0,387	0,317	0,189	0,028
	0,900	0,671	0,604	0,550	0,484	0,405	0,339	0,152	0,048
2,00	0,800	0,536	0,497	0,423	0,339	0,253	0,173	—	—
	0,900	0,594	0,507	0,452	0,344	0,289	0,185	0,091	—

$$\varkappa(\theta_{2p}\ \eta) = 1 + \eta\frac{1-\theta_{2p}(\tau)}{1-\theta^4_{2p}(\tau)}; \tag{26}$$

$$\text{Fo}^0 = \frac{at_0}{R^2} = \frac{\tau_0}{\rho^2} = \frac{1}{12(1+m)}. \tag{27}$$

The proposed solution is applicable to the range of variations for the criteria

$$\text{Sk} \leqslant 2.0;\ \text{Bi} \leqslant 4.0;\quad \eta = \frac{\text{Bi}}{\text{Sk}} \leqslant 2. \tag{28}$$

The reader will find the tables needed for the practical utilization of this solution in this article.

The specified values of Sk and η from Table 1 are used to determine the relative temperature $\theta_{1p}(\tau_0)$ for the surface of the body at the instant that regular heating begins. The corresponding function $\Omega(\tau_0)$ is determined from Table 2 according to the value of $\theta_{1p}(\tau_0)$. Then, considering (27), we calculate the magnitude of the function $\Omega(\tau)$ from (16) for any specified value of the Fourier number $\text{Fo} = \tau/\rho^2 = at/R^2$.

With $\Omega(\tau)$ taken from Table 1, we determine the corresponding value of the relative surface temperature $\theta_{2p}(\tau)$.

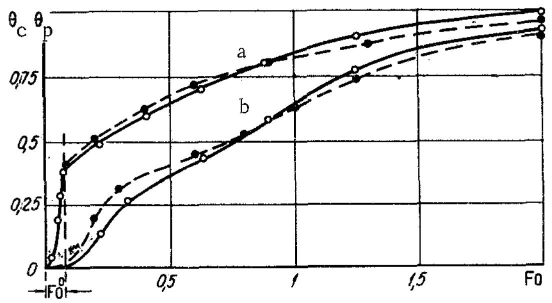

Fig. 1. Variation in the relative surface temperature θ_p (a) and in the midplane temperature θ_c (b) of a plate when $\eta = 1.0$; $\rho_{con} = 0.5$; Fo = 1/2.

If we know the function $\theta_{2p}(\tau)$ for the assumed values of Sk, η, and Fo, we use [14] to determine the relative temperature $\theta_2(\xi, \tau)$ at any point on the cross section of the body.

As an example, Fig. 1 in this article shows plots of the curves giving variations in the relative surface temperature $\theta_p(\tau)$ and in the midplane temperature $\theta_c(\tau)$ of a plate when Sk = 0.5 and $\eta = 1$. For purposes of comparison, the dashed lines show the corresponding curves derived by means of a computer [1].

From this comparison we can conclude that the proposed method is entirely suitable for the corresponding thermal calculation.

The method described here is favorably supported by its comparative simplicity and its accuracy, which is adequate for practical purposes.

NOTATION

$\theta(\xi, \tau) = T(x, t)/T_m$	is the relative temperature;
θ_0	is its initial value;
T_m	is the temperature of the heating medium;
$\beta(\tau) = h_a T_m^6 b(t)$;	
$\xi = h_a T_m^3 b(t)$;	
$l(t) = R - b(t)$	is the depth of penetration for the thermal perturbation;
2R	is the thickness of the plate or the diameter of the cylinder;
$\tau = a h_a^2 T_m^6 t$	is the dimensionless time;
m	is the shape factor for the body;
m = 0	is the plate;
m = 1	is the cylinder;
$h_a = \sigma_a/\lambda$	where σ_a is the apparent coefficient of radiation;
λ	is the coefficient of thermal conductivity;
α_{con}	is the heat-transfer coefficient for convection;
a	is the coefficient of thermal diffusivity;
$\eta = Bi/Sk$;	
$Bi = \alpha_{con}R/\lambda$	is the Biot number;
$Sk = h_a T_m^3 R$	is the Stark number;
$Fo = at/R^2$	is the Fourier number.

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